Elementary teachers' mathematical knowledge for teaching prerequisite algebra concepts

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Abstract

The current study investigated the effects of an undergraduate mathematics content course for pre-service elementary teachers. The participants' content knowledge was quantitatively measured using an instrument comprised of items from the Mathematical Knowledge for Teaching Measures (Hill, Schilling, & Ball, 2004). Using a one-group pretest-posttest design, matched pairs t-tests showed significant gains (p = .000) in both common and specialized content knowledge and in two areas of prerequisite algebra concepts (numbers and equations/functions). Results provide evidence of pre-service teachers developing mathematical understanding beyond common content knowledge within collegiate course settings.

Key Words: Pre-service Elementary Teacher, Mathematical Knowledge for Teaching, Algebra

Introduction

A great deal of attention is currently being given to the knowledge that is unique to the profession of teaching. There has been much discussion and consideration of the exact knowledge, both content and pedagogical, that teachers need to effectively teach their subject matter (Ball, Thames, & Phelps, 2008; Hill & Ball, 2009; Stylianides & Ball, 2004). In the field of mathematics, a large amount of this work has been done by the Learning Mathematics for Teaching (LMT) Project, where researchers have been working to create a framework for teacher knowledge and develop measures for testing its various domains (Ball, et al., 2008; Hill, Schilling, & Ball, 2004). This group has been using their developed Mathematical Knowledge for Teaching (MKT) Measures to explore when and where teachers develop various aspects of knowledge through teacher preparation and professional development opportunities and how this knowledge can affect student achievement (Hill & Ball, 2004; Hill, Rowan, & Ball, 2005). The LMT researchers believe that student achievement could ultimately benefit from the study of "whether and how different approaches to teacher development have different effects on particular aspects of teachers' pedagogical content knowledge" (Ball, et al., 2008, p. 405). The current study works towards that goal, by using the MKT measures to explore content knowledge development in the context of a collegiate elementary education preparation program.

Although elementary teachers are responsible for teaching a variety of mathematical topics, this study focuses on the content knowledge specifically necessary for teaching the concepts students need to master prior to entering the formal study of algebra. Algebra was chosen because of the significant role it can play in the mathematical and educational development of students. Due to the recent push for algebra for all, algebra is becoming an ever-larger factor in a student's ability to successfully finish high school and continue study at the collegiate level (Chazan, 2008).

Nonetheless, we continue to see poor levels of student algebra achievement in the U.S. (Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007).

Therefore, the current study was conducted to test the effect of an undergraduate mathematics content course for elementary teachers on pre-service teachers' knowledge needed for teaching prerequisite algebra concepts. We begin by looking at the importance of algebra and the mathematical concepts considered prerequisite for the subject's formal study. Next, we discuss the potential influence teacher knowledge can have on student learning and the facets of knowledge specifically needed by teachers of mathematics. Details on the current study's research design and methodology follow, including a brief description of the administered instrument's construction using MKT items. After a report of statistical results, we conclude with a discussion of research findings and implications.

Background Information

Algebra: Issues surrounding algebra preparation are of growing concern, as many states are moving towards increasing the number of years of mathematics required to graduate from high school (Reys, Dingman, Nevels, & Teuscher, 2007). The Center for the Study of Mathematics Curriculum reported that of the 25 states that outline specific mathematics courses required for a high school diploma, 19 states now require Algebra I. This trend has created a need for all students, no longer just the college-bound, to be algebra proficient (Achieve, 2007; Chazan, 2008). Despite the significant impact algebra can have on a student's ability to continue his or her education and pursue certain career options, the algebra achievement of U.S. students on the National Assessment of Educational Progress (NAEP) has been and continues to be poor (Chazan & Yerushalmy, 2003). In fact, on the 2005 NAEP, only 6.9% of U.S. 17-year-olds scored at or above proficiency on multistep problem solving and algebra (National Center for Education Statistics, 2005; cited in Ketterlin-Geller, et al., 2007).

To address this issue, researchers, teachers, and curriculum experts have worked to identify the prerequisite content areas believed to contribute to a student's ability to succeed in algebra. For example, the Southern Regional Education Board (SREB) produced a list of 12 algebra-specific skills, i. e. Readiness Indicators, to classify the necessary prior knowledge needed for success in Algebra I (Bottoms, 2003). The list was developed by experts in the field of mathematics education, but not research-based. Therefore, prior to the current study, we investigated the similarities and differences between both recent and historical research and the Readiness Indicators. Through this and continued analyses, eight concepts that are prerequisite to success in a first algebra course have been identified through supporting research: (1) numbers and numerical operations, (2) ratios/proportions, (3) the order of operations, (4) equality, (5) patterning, (6) algebraic symbolism including letter usage, (7) algebraic equations and functions, and (8) graphing (Welder, 2006). Since multiple prerequisite algebra concepts can be addressed simultaneously, areas of overlap were naturally condensed into two prerequisite algebra constructs:

1. *Number Concepts* involve the skills related to reading, writing, representing, and computing with numbers in a variety of forms, including integers, fractions, decimals, ratios, and proportions. Since correct usage of the order of operations is vital to numerical computations, this concept is also included in this construct.
2. *Equation/Function Concepts* entail a conceptual understanding of variables, in addition to an ability to express generalizations, represent situations algebraically, simplify and solve algebraic representations (including linear equalities and inequalities), use formulas, and understand the relationship between an equation and its graphical representation. These

> tasks require a proper understanding of algebraic symbolism, including an expanded interpretation of the plus and equal signs, and letter usage in algebra. Since teachers commonly use the analysis and generalization of patterns to introduce students to functional relationships, patterning ideas are also included under this construct.

The National Council of Teachers of Mathematics recommends all of the concepts identified above, as prerequisite to algebra, to be covered within the K-8 mathematics curriculum (National Council of Teachers of Mathematics, 2000). Therefore, although elementary teachers are generally not involved in the teaching of formal algebra, they are responsible for preparing students with the necessary background knowledge that will be needed for learning algebra. Consequently, if the future goals of algebra are to be achieved, elementary teachers must be effectively teaching prerequisite algebra concepts to their students.

Knowledge Needed for Effective Teaching: Although many factors affect a teacher's effectiveness, teacher knowledge is one of the biggest influences on classroom atmosphere and student achievement (Fennema & Franke, 1992). In a meta-analysis of 60 studies, variables such as teacher ability, knowledge, and education level were found to have positive effects on student achievement (Greenwald, Hedges, & Laine, 1996). Furthermore, the work of Hill, Rowan, and Ball (2005) showed that teachers with increased knowledge produced significantly larger gains in student achievement, even though they controlled for many other variables (including student socioeconomic status, student absence rate, teacher credentials, teacher experience, and average length of mathematics lessons). These studies highlight the importance of *teacher knowledge*; however, researchers have struggled to classify and clearly define all of the elements comprising the knowledge teachers needing for teaching (Thames & Ball, 2010).

Researchers agree that subject area knowledge is an essential aspect of teacher knowledge. In fact, Ball, Thames and Phelps (2008) have stated that there may be nothing more foundational to teacher competency than how well teachers know the subjects they teach. This is supported by Rech, Hartzell, and Stephens (1993) and Ma (1999) who argue that a profound understanding of fundamental mathematics provides a necessary base for successful mathematics teaching. According to Ball, Hill, and Bass (2005), the quality of mathematics teaching depends on teachers' mathematical content knowledge; and, alarmingly, many U.S. teachers lack firm mathematical understanding and skill.

While no one disputes that teachers need a thorough understanding of the subject matter they teach, the focus on teacher knowledge has been redirected, over the past twenty five or so years, to the additional types of content knowledge needed specifically by teachers (as compared to other professionals in their subject areas). Shulman (1986) was the first to concentrate on the role content plays in teacher knowledge. Prior to this work, content area was merely considered a context in which teachers used their general knowledge of teaching. Ever since Shulman introduced the idea of *content knowledge for teaching* as distinct from disciplinary content knowledge, researchers have been working to identify and categorize its various facets.

In the field of mathematics, significant contributions have been made by the Learning Mathematics for Teaching (LMT) Project (Ball, 2003; Ball, et al., 2008; Hill & Ball, 2004, 2009; Hill, et al., 2004). Their work has confirmed that general mathematical ability does not fully account for the knowledge and skills needed for effective mathematics teaching. They have exposed a special type of knowledge needed by teachers that is specifically mathematical, separate from pedagogy and knowledge of students, and not needed in other professional settings (Ball, et al., 2008; Hill, et al., 2004). This is because the daily tasks of teachers, interpreting someone else's work, representing and forging links between ideas in multiple forms, developing alternative explanations, and choosing usable definitions, require knowledge beyond that which is needed to reliably carry out a mathematical algorithm (Ball, 2003; Ball, et al., 2005; Hill & Ball, 2009). These types of responsibilities require *decompressed* or *unpacked* mathematical reasoning, in addition to pedagogical thinking, demanding teachers to know *more* and *different* mathematics than what is needed by other adults (Ball, et al., 2008). This mathematical knowledge and skill unique to teaching has been termed *specialized content knowledge* (SCK) (Hill & Ball, 2009, p. 400). On the other hand, *common content knowledge* (CCK) is that which allows a person to successfully solve mathematical problems in non-classroom contexts, including "being able to do particular calculations, knowing the definition of a concept, or making a simple representation" (Thames & Ball, 2010, p. 223).

For teachers to be prepared to teach quality mathematics, teacher educators must ensure that pre-service teachers have opportunities to develop the mathematical knowledge that is specific to their needs (Lee, Meadows, & Lee, 2003). "Improving the mathematics learning of every child depends on making central the learning opportunities of our teachers," (Ball, 2003, p. 9). Some undergraduate programs for the preparation and certification of elementary teachers address mathematical content by having their students take general mathematics courses, such as Calculus (Battista, 1994). However, Ball et al. (2008) note that the mathematical demands of teachers are rarely addressed within standard university mathematics courses. Through their work, they have concluded that certain aspects of mathematical knowledge (like SCK) need to be addressed in mathematics courses specifically designed for teachers.

Teacher educators must work to develop SCK, in addition to CCK, of pre-service teachers within collegiate course settings (Battista, 1994; Chen & Ennis, 1995; Davis & McGowen, 2001; Manouchehri, 1996; Miller, 1999; Stacey, et al., 2001). Typically undergraduate elementary education programs require, or at least offer, one or two mathematics content courses for teachers. However, even though these courses are specifically designed for pre-service elementary teachers, some focus solely on the enhancement of CCK. Since these courses may be the only opportunity pre-service teachers have to develop the mathematical content knowledge needed for teaching prior to entering the classroom, these courses need to address both the CCK of prerequisite algebra concepts and the SCK needed for teaching them.

Research Design and Methodology

This study investigated an undergraduate mathematics content course for elementary education majors and its ability to develop pre-service teachers' CCK and SCK of prerequisite algebra skills. Gains in mathematical content knowledge were examined through a pre-experimental one-group pretest-posttest design. A quantitative instrument was developed for measuring pre-service teachers' CCK and SCK of prerequisite algebra constructs. This instrument was then implemented to address the following research questions (both with respect to an undergraduate first-semester elementary education mathematics content course):

1. What effects does this course have on pre-service teachers' mathematical content knowledge of individual prerequisite algebra constructs (number concepts and equation/function concepts)?
2. What effects does this course have on pre-service teachers' CCK and SCK of prerequisite algebra concepts?

Sample: Pre-service elementary teachers were sampled from a public, mid-sized, land-grant university in the western United States. These students are required to take a yearlong sequence of elementary-specific mathematics content courses. The first semester of this sequence (hereafter denoted MATH I) addresses sets, whole numbers (operations, properties, and computations), number theory, fractions, decimals, ratios, proportions, percents, integers, and sometimes rational and real numbers (Musser, Burger, & Peterson, 2005). The curriculum of the second semester, contrastingly, focuses on geometry, statistics, and probability. Due to the varied aims of these courses, MATH I is the only content course that directly addresses any of the eight prerequisite algebra concepts (numbers and numerical operations, ratios/proportions, the order of operations, equality, patterning, algebraic symbolism including letter usage, algebraic equations and functions, and graphing). Furthermore, due to the variety of collegiate methods courses and experiences working with children afforded to students, this course could be the only exposure to prerequisite algebra concepts some pre-service teachers get before entering the teaching profession. Therefore, it is paramount that MATH I is successful in developing pre-service teachers' mathematical content knowledge (both CCK and SCK) of prerequisite algebra concepts. Thus, the current study was designed to examine the effectiveness of MATH 1 by sampling all students who completed the course, during one fall semester (n = 48). With only minor variations, these students were mostly female and freshmen of traditional age.

Course Design and Instruction: In order to enroll in MATH I, students have to first meet one of four requirements: (1) Successful completion of Introductory Algebra (or a higher level mathematics course) with a grade of D or better, (2) Successful completion of the university's mathematics placement exam (at the level which allows enrollment in College Algebra or higher), (3) ACT math score of 23 or higher, or (4) SAT math score of 530 or higher. MATH I is a four-credit semester course that meets for 50-minute time periods, four days a week, for approximately 16 weeks. Three sections of the course were offered during the semester of data collection. Throughout this course, material was examined through a variety of instructional strategies including lecture, class discussion, hands-on activities, group-work and student collaboration, student presentations, writing tasks, quizzes, and exams. Two of the three course sections were independently taught by doctoral graduate teaching assistants, while the third, as well as course supervision, was handled by an assistant professor of mathematics education. The three instructors met one hour once a week to design course activities and exams and to align course schedules and goals. Although the instructors wrote individual quizzes, the syllabus, most activities, and all exams were identical across the three sections. As stated on the course syllabus, course objectives are to:

1. Solve mathematical problems based on Polya's model and using a variety of strategies.
2. Identify the structure of the whole, integer, rational, and real number systems.
3. Perform mathematical operations in base ten and other bases, use traditional and alternative algorithms, and solve elementary problems in number theory and set theory.
4. Apply technology appropriately in exploring and solving mathematical problems.
5. Model and use an activity-oriented approach to teaching and learning mathematics.
6. Encourage discourse, self-motivation, and independent thinking in learning mathematics.

The MATH I course curriculum, which is considered standard for this type of mathematics content course offered for pre-service elementary teachers, sequentially followed Chapters 1-9 of the textbook, *Mathematics for Elementary Teachers: A Contemporary Approach, 7th edition* (Musser, et al., 2005). These nine chapters address: (1) Problem Solving, (2) Sets, Whole Numbers, and Numeration, (3) Whole Numbers: Operations and Properties, (4) Whole-Number Computation – Mental, Electronic, and Written, (5) Number Theory, (6) Fractions, (7) Decimals, Ratio, Proportion, and Percent, (8) Integers, and (9) Rational Numbers and Real Numbers, with an Introduction to Algebra. During the semester of data collection, the MATH I curriculum deviated only slightly from the sequence outlined in this textbook: Section 4.3 (Algorithms in Other Bases) was eliminated, Sections 2.4 (Relations and Functions) and 9.3 (Functions and Their Graphs) were combined and covered together after Chapter 3, and the order of Sections 9.1 (The Rational Numbers) and 9.2 (The Real Numbers) were reversed. Additional student activities and supplementary materials used in the course came from multiple sources, including Dolan, Willianson, and Muri (2007), Friel, Rachlin, and Doyle (2001), Johnston (1998), Lappen, Fey, Fitzgerald, Friel, and Phillips (1998), Willard (unpublished activity, 2006), and Williams and Bright (1998). Instructors had no knowledge of the items being tested by this study; therefore, instruction was completely disconnected from the administered instrument.

Instrument Construction: For this study, items from the Mathematical Knowledge for Teaching (MKT) Measures were used to design an instrument specifically to measure pre-service teachers' CCK and SCK of prerequisite algebra concepts. In developing the MKT Measures, the LMT Project created a question bank that contains hundreds of multiple-choice items designed to measure various facets of teachers' knowledge in the content areas of (1) number concepts and operations, (2) patterns, functions and algebra, and (3) geometry. (Ball, et al., 2005; Ball, et al., 2008; Hill & Ball, 2004; Hill, et al., 2005; Hill, et al., 2004). Some LMT items address CCK, such as, "What is the number that lies between 1.1 and 1.11?" (Ball, et al., 2008, p. 399). However, other items are written to target SCK, i.e. the mathematical ideas specific to the needs of teachers (Hill & Ball, 2004). The appendix contains two examples of quantitative items that measure SCK. These examples are from a small set of items that the LMT Project has released for public use (Learning Mathematics for Teaching Project, 2008).

Items were chosen from the LMT question bank to create an instrument that could adequately measure four individual constructs: CCK, SCK, number concepts, and equation/function concepts. The LMT researchers used item response theory to create information for each of the question bank items from their various pilot studies; this information was used to guide item selection. Items were chosen based on three criteria: (1) the construct addressed by the item (only questions regarding CCK or SCK of numbers or equations/functions were considered), (2) the difficulty of the item (to assure that the overall difficulty of the instrument was appropriate for the targeted population), and (3) the amount of information the item could provide about the participants (to minimize the number of questions on the instrument). A total of 51 items were needed to produce an instrument with optimal testing abilities across all four constructs (CCK, SCK, number concepts, and equation/function concepts). The embedded (yet overlapping) measures resulted in the following number of items: numbers: 29; equations/functions: 23; CCK: 31; SCK: 20 (see Welder (2007) for additional detail regarding instrument construction).

Instrument Administration and Scoring: During the first week of the semester, all MATH I students enrolled in three course sections (n = 69) were asked to complete the instrument, as a pretest measure of their CCK and SCK of prerequisite algebra concepts. Although students were not given incentives to participate and completion of the instrument had no bearing on their course grades, all but one student present on the day of pretest data collection chose to participate in the study (n = 68). The students worked on the instrument during one of their class meetings, for 35-50 minutes. Each student received a total of four pretest scores reporting the number of correct responses provided for each of the four individual constructs (CCK, SCK, number concepts, and equation/function concepts). Pretest scores were used to calculate Cronbach's alphas to test internal reliability of the overall instrument and each of the four embedded measures. All were shown to be reliable based on the LMT designated criterion of $\alpha \geq .70$ for sample sizes greater than 60.

The instrument was then administered to the same set of students a second time, during the last week of the semester. LMT does allow for one form of an MKT instrument to be used as both pretest and posttest, as long as the administrations are separated by at least three months, to minimize test-retest effects. The administration dates for this study were carefully chosen to ensure that this requirement was met. All students present on the day of posttest administration (n = 54) participated and, again, most worked for 35-50 minutes. A total of 48 students completed both administrations of the instrument and were therefore included in the sample examined to answer the research questions of this study (n = 48).

MKT instruments are designed so that typical teachers correctly answer 50% of the items; and, the results of this study did reflect this percentage. However, because of this design, the LMT researchers discourage reporting raw scores and percentages because they may mislead the public about teachers' overall level of knowledge. Therefore, all raw scores were standardized according to the statistics calculated from the pretest scores for each measure. Once all raw pretest scores were standardized to z-scores, a raw score to z-score conversion table was constructed for each measure. These tables were then used to standardize the raw scores resulting from the posttest.

Statistical Analysis: To explore gains in CCK, SCK, and knowledge of prerequisite algebra constructs (numbers and equations/functions), a matched pairs t-test (t) was used to compare standardized pretest and posttest total scores within the single sample for each construct. For example, to examine growth in knowledge of number concepts, the difference in each student's standardized pretest and posttest number scores was calculated. The hypothesis claiming that the true mean difference in standardized pretest and posttest number scores is equal to zero was then tested against a two-sided hypothesis for a non-zero population mean difference. This process was repeated for each of the other three constructs: equations/functions, CCK, and SCK.

Statistical Results

The mean standardized difference in pretest and posttest number scores within the sample was .7889, indicating that students' knowledge of number concepts improved an average of .7889 pretest standard deviations ($t(47) = 7.810$, $p < .001$). For equation/function scores, the mean standardized difference was much smaller, but still significant, at .3906 pretest standard deviations ($t(47) = 4.704$, $p < .001$). Significant growth was also found in both students' CCK and SCK. Improvement in these areas averaged .5431 ($t(47) = 6.192$, $p < .001$) and .6438 ($t(47) = 5.198$, $p < .001$) pretest standard deviations respectively (see Table 1 for summary statistics).

Since students' knowledge of number concepts improved an average of .7889 pretest standard deviations, this translates to the students correctly answering an average of 3.54 more of the 29 number items on the posttest instrument versus the pretest. Similarly, results indicate that on average the students also correctly answered an additional 1.63 of the 23 equation/function items, 2.75 of the 31 CCK items, and 2.33 of the 20 SCK items.

The confidence intervals ($\alpha = .05$) for true mean improvement in each construct were also converted to create confidence intervals for the true mean increase in the percentage of items answered correctly. These results indicate that the mean percentage increase for correctly answered number items lies between 9.07% and 15.36%. For equation/function items, the percentage of correctly answered items increased between 4.04% and 10.09%. These percentages fall between 6.00% and 11.74% and between 7.15% and 16.18% for CCK and SCK respectively.

Table 1
Summary of t-test results

	Standardized Difference in Number Scores	Standardized Difference in Equation/Function Scores	Standardized Difference in CCK Scores	Standardized Difference in SCK Scores
t-test	$t_N = 7.810$	$t_E = 4.704$	$t_C = 6.192$	$t_S = 5.198$
p-value	$p < .001^*$	$p < .001^*$	$p < .001^*$	$p < .001^*$
Sample Mean Difference	$\overline{x}_N = .7889$	$\overline{x}_E = .3906$	$\overline{x}_C = .5431$	$\overline{x}_S = .6438$
95% Confidence Interval for True Mean Difference	$\mu_N \in (.5857, .9921)$	$\mu_E \in (.2236, .5577)$	$\mu_C \in (.3667, .7196)$	$\mu_S \in (.3946, .8930)$

* Result was significant using $\alpha = .05$.

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Discussion

Number Concepts: Mastery of number concepts and numerical operations are fundamental to a student's ability to learn algebra (Baroudi, 2006; Booth, 1984, 1986; Christou & Vosniadou, 2005; Christou, Vosniadou, & Vamvakoussi, 2007; Gallardo, 1995, 2002; Kieran, 1985; Rotman, 1991; Tent, 2006; Wu, 2001). Difficulties of elementary algebra students are often caused by a misunderstanding of surrounding numerical computational ideas, including inverse operations, associativity, commutativity, distributivity, and the order of operations (Booth, 1984). It is essential that students understand and be able to fluently use these and other basic number and operation properties for algebraic manipulation and equation solving (Watson, 1990). Therefore, the instrument for this study was constructed to address several of the important number and operation concepts that are deemed prerequisite for the study of formal algebra. Items were specifically chosen to assess knowledge of whole number operations, subtraction of integers, representations and explanations of fractions and fraction operations, decimal representations, prime numbers, and the order of operations.

The largest increase noted in pre-service teachers' mathematical content knowledge, after the completion of MATH I, was in the construct of numbers and numerical operations. This result is encouraging, although not surprising, since the MATH I course curriculum is designed to focus on numbers and numerical operations. This finding is therefore consistent with the construction of MATH I and shows that the course improves pre-service teachers' knowledge of the content it purports to teach.

Equation/Function Concepts: The functional relation between two variables is a central concept in prealgebra courses. According to Brenner et al. (1995) and Brenner et al. (1997), translating and applying mathematical representations of functional relations are cognitive skills essential for algebraic reasoning. It is therefore encouraging to see significant growth, although it was the smallest amount of growth recorded, in pre-service teachers' knowledge of equation/function concepts after their completion of MATH I. These results could be considered surprising, given that the MATH I curriculum focuses substantially on numbers and numerical operations, as opposed to equations and functions. In fact, only three 50-minute class sessions were dedicated to the topic. The students spent two sessions discussing relations and functions and a third session cooperatively completing an unpublished activity, "Fun with Functions!" by Willard (2006) (which asks students to represent functional relationships, described with words, using symbols, tables, and graphs). In addition to the students having only a brief encounter with functions and relations throughout this course, this content was addressed during the end of September, almost eleven weeks prior to the posttest administration of the instrument. Given these two facts, the researchers did not expect to see significant growth in the content area of equation and functions.

One possible explanation for this growth could be that this course helped reacquaint students with material they have encountered in the past but may have since forgotten. For many of these students, MATH I is not only their first collegiate mathematics course, but the first mathematics course they have taken since their sophomore or junior year of high school. Perhaps it is merely the review of mathematical content that is leading to the observed growth in the knowledge of equations and functions.

Common Content Knowledge: When the test items were analyzed in terms of the *type* of content knowledge they measured as opposed to the content *area* they addressed, the researchers also found significant growth in the pre-service teachers' CCK after completing MATH I. The content covered in this course's curriculum does not exceed that which is typically covered in K-8 classrooms. Furthermore, CCK only requires that a person possess the skills and procedures necessary for solving (not explaining or representing) mathematical problems. Therefore, the items used to measure CCK in this study, only tested the pre-service teachers' abilities to *solve* K-8 level mathematics problems (pertaining to prerequisite algebra concepts). It was assumed that students had substantial mastery of this level of computational mathematics prior to taking MATH I, due to the course prerequisites. This, however, appears not to be true, given that the pre-service teachers in this study showed significant growth in CCK after course completion. This finding supports existing literature that has identified gaps in the mathematical knowledge of elementary teachers and suggests that the mathematical content knowledge of pre-service and inservice elementary teachers may be insufficient (Ball, 1988, 1990; Ball & Wilson, 1990; Ilany, Keret, & Ben-Chaim, 2004). Therefore, this study reinforces the need for courses such as this one, heavy in content and dedicated to K-8 mathematical content, designed specifically for future elementary teachers.

Specialized Content Knowledge: In the field of mathematics, how teachers hold knowledge may matter more than how much knowledge they hold (Hill & Ball, 2004). In fact, "teaching quality might not relate so much to performance on standard tests of mathematics achievement as it does to whether teachers' knowledge is procedural or conceptual, whether it is connected to big ideas or isolated into small bits, or whether it is compressed or conceptually unpacked" (Hill & Ball, 2004,

p. 332). Researchers assert that this *additional* mathematical knowledge required of teachers (or lack thereof) can impact instructional decisions and ultimately the achievement of their students (Ball & Wilson, 1990; Graeber, 1999; Lee, et al., 2003; Rine, 1998). In fact, Hill et al. (2008) found teachers' mathematical knowledge for teaching to be strongly related to "the mathematical quality of their instruction, including their use of mathematical explanation and representation, responsiveness to students' mathematical ideas, and ability to avoid mathematical imprecision and error" (Hill & Ball, 2009, p. 70).

Despite the significance of teacher knowledge, university mathematics courses rarely address the mathematical demands of teachers (Ball, et al., 2008). Battista (1994) urges for teacher education institutions to offer multiple mathematics courses, specifically for teachers, that treat mathematics as "sense making" rather than "rule following". He believes that pre-service elementary teachers will not be adequately prepared to teach mathematics by simply taking more college-level mathematics courses. Recent findings of the National Mathematics Advisory Panel (National Mathematics Advisory Panel, 2008) support this assertion by showing that achievement gains of elementary students in mathematics could not be predicted by the number and level of mathematics courses taken by their teachers. Teachers must be taught mathematics properly before they can be expected to teach it properly; yet, most university mathematics courses merely reinforce the view of mathematics as a set of memorized procedures (Battista, 1994). Therefore, taking more of these courses will most likely do nothing to enhance teachers' SCK. Instead, SCK has to be addressed in content courses specifically designed for pre-service elementary teachers (Ball, et al., 2008). Unfortunately, though, even these courses often do not have the time or concentration required to develop the specialized mathematical knowledge needed by elementary teachers (Battista, 1994). Ball (2003) agrees, stating that "few mathematics courses offer opportunities to learn mathematics in ways that would produce such knowledge" (p. 8). However, the current study identified significant growth in the SCK of pre-service elementary teachers upon their completion of MATH I. These results are very encouraging and show promise for the future of teacher education in this area.

Scholars once believed that the best way for a person to acquire any aspect of pedagogical content knowledge was through practical experience as a teacher. In fact, collegiate teacher education was once thought to be incapable of making significant contributions to the unique knowledge needed for teaching (Ball & Wilson, 1990). However, the work of Davis and McGowen (2001) demonstrated that collegiate content courses for teachers do have the potential to enhance content knowledge specific to teachers. In this work, they followed one pre-service elementary teacher and showed how her mathematical understanding evolved significantly while she was taking a collegiate mathematics course. The findings of the current study further challenge the claims of Ball and Wilson (1990) by extending Davis and McGowen's valuable research beyond a singular case study, to show that similar results can be achieved with larger groups. Therefore, this study provides evidence in support of the numerous scholars who argue that it is not only necessary, but in fact possible, for teacher educators to develop pre-service teachers' pedagogical content knowledge within collegiate course settings (Battista, 1994; Chen & Ennis, 1995; Davis & McGowen, 2001; Manouchehri, 1996; Miller, 1999; Stacey, et al., 2001).

Implications

1. *Content courses should include topics related to the study of equations and functions.* If elementary teachers are to successfully prepare students to learn algebra, they need to be properly prepared to teach fundamental concepts surrounding the ideas of equations and functions. This

study suggests that dedicating even as few as three 50-minute class sessions to the study of functions and relations can significantly improve pre-service teachers' knowledge of these topics. These results encourage teacher educators to dedicate time in their content courses for elementary teachers to address ideas surrounding the study of equations and functions. However, this is not to imply that 150 minutes is a recommended or sufficient amount of time to spend on these topics.

2. *Content and pedagogical ideas and practices may be blended together to enhance both common and specialized content knowledge simultaneously.* The significant growth of SCK identified through this study supports the structure and delivery of this particular mathematics content course for elementary teachers. Determining the exact aspects of the MATH I course that promoted the development of SCK is beyond the scope of this study. However, it is speculated that the integration of methodology into this content course may have been a contributing factor. The course instructors incorporated instructional and pedagogical strategies through the use of various materials, manipulatives, and hands-on activities. For example, the pre-service teachers were exposed to student thinking and common errors through analyzing actual examples of K-8 student work. It can be very time-consuming to integrate aspects of mathematics teaching that are typically considered to be methodology into content courses, like MATH I, and some instructors debate the time tradeoff. However, the results of this study support the continuation of these practices, since MATH I students developed knowledge beyond that which is considered CCK. Furthermore, this course was capable of enhancing *both* types of content knowledge (CCK and SCK) simultaneously, suggesting that the students' CCK was not compromised by this practice. Although additional research is needed to confirm this assertion, the current study suggests possible incentives for blending together the ideas and practices that are generally divided between mathematics content and methodology courses.

3. *Course prerequisites need to be implemented and enforced.* The significant increase found in the CCK of MATH I students indicates that these pre-service teachers had not mastered computational skills of K-8 level prerequisite algebra concepts prior to taking this class. Course prerequisites were outlined in the syllabus; however, during the semester under investigation, they were not strictly enforced. This may have influenced the level of mathematical knowledge students had upon entering this course and may explain the alarming growth observed in their CCK. Therefore, the findings of this study recommend the implementation and enforcement of appropriate prerequisites for mathematics content courses for pre-service elementary teachers.

4. *Further research is required to identify, understand, and replicate factors affecting growth in specialized content knowledge.* Although research has been working to detect if and where teachers' SCK is forming, there is not yet an explicit understanding of *how* this knowledge is developed. The current study was able to show that growth in SCK occurred throughout an undergraduate mathematics content course, yet it is unclear exactly what aspects of the MATH I course helped these students specifically acquire this knowledge. Researchers now need to analyze the content and pedagogy of courses and programs that are successful in building SCK to better understand *how* these opportunities are supporting SCK development.

Summary

As more states move towards increasing the number of years of mathematics required to graduate from high school, the number of students needing to successfully complete a course in algebra to receive a high school diploma is undoubtedly rising (Achieve, 2007; Reys, et al., 2007). This trend highlights a growing need for elementary and middle school teachers (K-8) to be effectively preparing students for the formal study of algebra. Research suggests that for students to succeed in their first algebra course they must master both number and equation/function concepts throughout their K-8 mathematics education (Welder, 2006). Furthermore, research has illustrated that student achievement can be affected by teachers' knowledge (Fennema & Franke, 1992; Greenwald, et al., 1996; Hill, et al., 2005). It can therefore be argued that the development of elementary and middle school teachers' knowledge of prerequisite algebra concepts is an important key to ensuring the future algebra success of students. According to the framework put forth by the LMT Project, the Mathematical Knowledge for Teaching consists of mathematical content knowledge specific to the needs of teachers (SCK), in addition to that which is considered common for other professionals (CCK) (Ball, et al., 2008; Hill & Ball, 2004, 2009; Hill, et al., 2004; Thames & Ball, 2010). Therefore, teacher educators can support student algebra achievement by ensuring that K-8 teachers are provided with opportunities to develop both CCK and SCK for effectively teaching prerequisite algebra concepts.

The current study found that pre-service elementary teachers significantly improved both their CCK and SCK in the areas of numbers and equations/functions after completing a one-semester, undergraduate mathematics content course for teachers. Growth in knowledge of equations and functions was particularly interesting because this content is, for the most part, considered to be outside the scope of the course curriculum. Furthermore, these results highlighted opportunity for improvement in pre-service teachers' CCK of elementary and middle school level mathematics content. These findings support the need for content courses for pre-service elementary teachers, such as this one, to specifically address the mathematical content that teachers at these levels will be expected to teach. Perhaps most importantly, this work validates the ability of collegiate teacher preparation courses to develop mathematical knowledge specific to the needs of teachers. In particular, we have shown that an undergraduate content course can have a significant effect on pre-service elementary teachers' SCK of mathematics. This finding encourages the efforts of teacher educators and calls for researchers to extend work in this area to explore the elements of this (and other) learning opportunities that are promoting the acquisition of SCK. If we can identify the particular content and/or pedagogical components that are supporting this learning, we can begin to develop interventions for teacher preparation programs and professional development opportunities to explicitly build and strengthen teachers' SCK. Ultimately, teacher educators can work to facilitate the algebra achievement of students, through the development of elementary teachers' knowledge needed for teaching prerequisite algebra concepts.

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Appendix

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x 25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do <u>not</u> represent multiplication of fractions appropriately.

Which model below <u>cannot</u> be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)