

## **Prerequisite Knowledge for the Learning of Algebra**

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### **Abstract**

Researchers, teachers, and curriculum experts have noted content areas believed to contribute to students' abilities to succeed in algebra. Specifically, the Southern Regional Education Board (SREB) produced a list of 12 algebra-specific skills, Readiness Indicators, which classify the prior knowledge necessary for success in Algebra I (Bottoms, 2003). The list was developed by mathematics education experts, but not based upon research. Therefore, the current investigation explores similarities and differences between the relevant research and the Readiness Indicators. Research indicates that prior to learning algebra, students must have an understanding of numbers, ratios, proportions, the order of operations, equality, algebraic symbolism (including letter usage), algebraic equations and functions. These results partially, if not fully, support 8 of the 12 Readiness Indicators.

### Prerequisite Knowledge for the Learning of Algebra

On the first day of an Algebra I course, a teacher is presented with an entirely new class of students, all with varying backgrounds and differing degrees of preparation. Where do these teachers begin? How do they know if their newest group of students is prepared to begin learning the content of an algebra course? Is there prerequisite content that should be reviewed, or perhaps even covered for the first time, to properly prepare these students for the material they are about to encounter? What content is prerequisite for the learning of algebra?

This question takes on major significance when one views the importance of algebra to a student's secondary and post-secondary education. Algebra anchored its existence in the secondary school mathematics curriculum after it became a college entrance requirement at Harvard University in 1820 (Rachlin, 1989). Ever since, algebra has had the ability to determine the educational opportunities available to college-intending students (Moses, 1994; Moses, Kamii, Swap, & Howard, 1989; Picciotto & Wah, 1993). Algebra can separate people from further progress in mathematics-related fields of study (Davis, 1995). However, more and more districts and states have added algebra to their high school graduation requirements causing the need for all students, no longer just the college-bound, to be algebra proficient. For the students who do continue their education, algebra concepts are prerequisite for studying every branch of mathematics, science, and technology (Fey, 1989).

Despite the significance of algebra in students' educations and futures, the algebra achievement of U.S. students on the National Assessment of Educational Progress (NAEP) is poor (Chazan & Yerushalmy, 2003). In fact, 53.8% of all responses given on Remedial Intermediate Algebra exams by a group of freshman college students were incorrect (Pinchback, 1991). Pinchback categorized an alarming 40.2% of these incorrect responses as result of errors

caused by lack of prerequisite knowledge. Could poor algebra achievement be due to students not being properly prepared for algebra courses? If lack of preparation is a problem, then it is essential to identify content whose mastery is required for the learning of algebra. What do students need to know prior to entering an algebra course in order to be successful?

### Defining Algebra

According to Booth (1986), the main purpose of algebra is to learn how to represent general relationships and procedures; for through these representations, a wide range of problems can be solved and new relationships can be developed from those known. However, students tend to view algebra as little more than a set of arbitrary manipulative techniques that seem to have little, if any, purpose to them (Booth, 1986). Perhaps the typical algebra curriculum focuses too heavily on simplification and manipulation, rather than the generalized ideas that create the basis of algebra. Interestingly, the content included in high school algebra has changed very little over the past century (Kieran, 1992).

Standard first-year algebra classes generally include: operations with positive and negative numbers; evaluation of expressions; solving of linear equations, linear inequalities and proportions; age, digit, distance, work and mixture word problems; operations with polynomials and powers; factoring of trinomials, monomial factoring, special factors; simplification and operations with rational expressions; graphs and properties of graphs of lines; linear systems with two equations in two variables; simplification and operations with square roots; and solving quadratic equations (by factoring and completing the square) (Usiskin, 1987). More concisely, high school algebra can be outlined in five major themes (a) variables and simplification of algebraic expressions, (b) generalization, (c) structure, (d) word problems, and (e) equations (Linchevski, 1995). How do teachers prepare students for learning such ideas?

## Readiness Indicators

In one of the most recent papers addressing the issue of identifying prerequisite knowledge for the learning of algebra, the Southern Regional Education Board (SREB) created a panel of classroom teachers and curriculum experts (from the Educational Testing Service) to analyze curriculum materials. Using their professional expertise, members worked cooperatively to identify 17 mathematical concepts, named Readiness Indicators, believed to classify the skills necessary for a student to be successful in learning Algebra I. The first 5 Readiness Indicators address general processing skills vital to learning all mathematics. The next 12 Readiness Indicators, however, are content-specific to the learning of algebra (Bottoms, 2003) and are therefore most pertinent to the research topic at hand.

1. Read, write, compare, order and represent in a variety of forms: integers, fractions, decimals, percents, and numbers written in scientific and exponential notation.
2. Compute (add, subtract, multiply and divide) fluently with integers, fractions, decimals, percents, and numbers written in scientific notation and exponential form, with and without technology.
3. Determine the greatest common factor, least common multiple and prime factorization of numbers.
4. Write and use ratios, rates and proportions to describe situations and solve problems.
5. Draw with appropriate tools and classify different types of geometric figures using their properties.

6. Measure length with appropriate tools and find perimeter, area, surface area and volume using appropriate units, techniques, formulas and levels of accuracy.
7. Understand and use the Pythagorean relationship to solve problems.
8. Gather, organize, display and interpret data.
9. Determine the number of ways an event can occur and the associated probabilities.
10. Write, simplify and solve algebraic equations using substitution, the order of operations, the properties of operations and the properties of equality.
11. Represent, analyze, extend and generalize a variety of patterns.
12. Understand and represent functions algebraically and graphically.

#### The Problem

The SREB developed the list of Readiness Indicators through much deliberation among experts in the field of mathematics education, however they were not based upon results of research (Bottoms, 2003). Do current research findings correspond with the conclusions of the SREB? Which Readiness Indicators are supported by research? Which are not?

In an effort to answer these questions, the researcher reviewed literature addressing prerequisite knowledge for the learning of algebra, as well as misconceptions of algebra students. Although the latter does not directly identify prerequisite knowledge, research regarding deficiencies and difficulties of algebra students can provide insight into areas where algebra students are unprepared. Therefore, this type of research is considered relevant to the discussion of prerequisite knowledge. Algebraic content areas categorize the following discussion of literature.

## Analysis of Literature

### *Vocabulary*

Miller and Smith (1994) identified prerequisite vocabulary for the learning of algebra, due to their belief that lack of prerequisite vocabulary contributes to students' failure to retain problem-solving skills learned in previous mathematics courses. By reviewing course textbooks and interviewing mathematics instructors, they created a 60-item list of vocabulary terms deemed prerequisite for Intermediate Algebra and College Algebra students. This list was then narrowed to 30 items, with the assistance of 44 college mathematics professors from 19 different colleges. The selected vocabulary includes geometric terms such as perimeter, area, volume, and radius, as well as more traditional algebraic terms such as factor, linear equation, slope, and y-intercept. Miller and Smith (1994) then investigated Intermediate and College Algebra students' vocabulary, by administering a multiple-choice and true-false vocabulary test; students knew an average of 15 of the 30 terms.

### *Numbers*

Other researchers have focused on the value of understanding numbers prior to algebra introduction (Booth, 1984, 1986; Gallardo, 2002; Kieran, 1988; Rotman, 1991; Wu, 2001). According to Watson (1990), a better understanding of number basics would give students a stronger ability to handle algebraic operations and manipulations. What types of numbers need to be studied prior to learning algebra? The SREB's Readiness Indicator number 1 focuses on students' ability to read, write, compare, order, and represent a variety of numbers, including integers, fractions, decimals, percents, and numbers in scientific notation and exponential form (Bottoms, 2003). Some of these forms have also been mentioned in research addressing prerequisite number knowledge for the learning of algebra.

Gallardo (2002) focused on the fact that the transition from arithmetic to algebra is where students are first presented with problems and equations that have negative numbers as coefficients, constants and/or solutions. Therefore, she believes that students must have a solid understanding of integers in order to comprehend algebra. Lack of this understanding will affect students' abilities to solve algebraic word problems and equations. However, Gallardo's research showed that 12- and 13-year-old students do not usually understand negative numbers to the fullest extent.

Misconceptions of negative numbers were identified in earlier research done by Gallardo and Rojano (1988; cited in Gallardo, 2002) while investigating how 12- and 13-year-old beginning algebra students acquire arithmetic and algebraic language. One major area of difficulty involved the nature of numbers. Specifically, students had troubles conceptualizing and operating with negative numbers in the context of prealgebra and algebra. Therefore, Gallardo (2002) argues that while students are learning the language of algebra, it is imperative that they understand how the numerical domain can be extended from the natural numbers to the integers.

Kieran (1988) also found misunderstandings regarding integers to affect the success of algebra students in grades 8-11. During interviews with Kieran, students who had taken at least one year of algebra made computational equation-solving errors involving the misuse of positive and negative numbers. Furthermore, when these students were required to use division as an inverse operation, they tended to divide the larger number by the smaller, regardless of the division that was actually required within the operation. Therefore, students' errors extended into the division of integers, which implies a lack of understanding of fractions.

An opinion article regarding how to prepare students for algebra further supports the inclusion of fractions as prerequisite knowledge for the learning of algebra. According to

Wu (2001), fraction understanding is vital to a student's transformation from computing arithmetic calculations to comprehending algebra. Wu believes that K-12 teachers are not currently teaching fractions at a deep enough level to prepare students for algebra. In fact, she believes that the study of fractions could and should be used as a way of preparing students for studying generality and abstraction in algebra.

Fractions were also stressed when Rotman (1991) chose number knowledge as a prerequisite arithmetic skill for learning algebra. During a research project that mounted evidence against the assumption that arithmetic knowledge is prerequisite for successful algebra learning, Rotman constructed a list of arithmetic skills he considers as prerequisite to algebra. Based on his experiences as a teacher, Rotman argues that algebra students need to understand the structure behind solving applications, the meaning of symbols used in arithmetic, the order of operations and basic properties of numbers (especially fractions). Of course, in order to operate with fractions students are required to know basic number theory ideas including least common multiple. Therefore, the necessity of fraction knowledge partially supports Readiness Indicator number 3, which states that students need to be able to determine the greatest common factor, least common multiple, and prime factorization of numbers (Bottoms, 2003).

### *Proportionality*

Fractions commonly appear in beginning algebra in the form of proportions, which provide wonderful examples of naturally occurring linear functions. Because of this, Post, Behr, and Lesh (1988) feel that proportionality has the ability to connect common numerical experiences and patterns, with which students are familiar, to more abstract relationships in algebra. Proportions can also be used to introduce students to algebraic representation and variable manipulation in a way that parallels their knowledge of arithmetic.

In fact, proportions are useful in a multitude of algebraic processes, including problem solving, graphing, translating and using tables, along with other modes of algebraic representation. Due to its vast utility, Post et al. (1988) consider proportionality to be an important contributor to students' development of pre-algebraic understanding. Similarly, Readiness Indicator number 4 focuses on the importance of ratios, rates and proportions in the study of algebra (Bottoms, 2003).

Proportional reasoning requires a solid understanding of several rational number concepts including order and equivalence, the relationship between a unit and its parts, the meaning and interpretation of ratio, and various division issues (Post et al., 1988). Therefore, these concepts could be considered, along with proportional reasoning, prerequisite knowledge for the learning of algebra.

### *Computations*

In addition to understanding the properties of numbers, algebra students need to understand the rules behind numerical computations, as stated in Readiness Indicator number 2 (Bottoms, 2003). Computational errors cause many mistakes for algebra students, especially when simplifying algebraic expressions. Booth (1984) claims elementary algebra students' difficulties are caused by confusion surrounding computational ideas, including inverse operations, associativity, commutativity, distributivity, and the order of operations convention. These misconstrued ideas are among basic number rules essential for algebraic manipulation and equation solving (Watson, 1990). The misuse of the order of operations also surfaced within an example of an error made by collegiate algebra students that Pinchback (1991) categorized as result of lack of prerequisite knowledge. Other errors deemed prerequisite occurred while adding

expressions with radical terms and within the structure of long division (while dividing a polynomial by a binomial) (Pinchback, 1991).

Mentioned by Rotman (1991) as a prerequisite arithmetic skill, the order of operations is also included in Readiness Indicator number 10 (Bottoms, 2003). In fact, this convention has been found to be commonly misunderstood among algebra students in junior high, high school, and even college (Kieran, 1979, 1988; Pinchback, 1991). The order of operations relies on bracket usage; however, algebra requires students to have a more flexible understanding of brackets than in arithmetic. Therefore, according to Linchevski (1995), prealgebra should be used as a time to expand students' conceptions of brackets.

Kieran (1979) investigated reasons accounting for the common misconception of the order of operations and alarmingly concluded that students' issues stem from a much deeper problem than forgetting or not learning the material properly in class. The junior high school students, with which Kieran worked, did not see a need for the rules presented within the order of operations. Kieran argues that students must develop an intuitive need for bracket application within the order of operations, before they can learn the surrounding rules. This could be accomplished by having students work with arithmetic identities, instead of open-ended expressions.

Although teachers see ambiguity in solving an open-ended string of arithmetic operations, such as  $2 + 4 \times 5$ , students do not. Students tend to solve expressions based on how the items are listed, in a left-to-right fashion, consistent with their cultural tradition of reading and writing English. Therefore, the rules underlying operation order actually contradict students' natural way of thinking. However, Kieran suggests that if an equation such as  $3 \times 5 = 15$  were

replaced by  $3 \times 3 + 2 = 15$ , students would realize that bracket usage is necessary to keep the equation balanced (Kieran, 1979).

### *Equality*

Kieran's (1979) theory assumes that students have a solid understanding of equations and the notion of equality. Readiness Indicator number 10 suggests that students are familiar with the properties of equality before entering Algebra I (Bottoms, 2003). However, equality is commonly misunderstood by beginning algebra students (Falkner, Levi, & Carpenter, 1999; Herscovics & Kieran, 1980; Kieran, 1981, 1989). Beginning algebra students tend to see the equal sign as a procedural marking that tells them "to do something," or as a symbol that separates a problem from its answer, rather than a symbol of equivalence (Behr, Erlwanger, & Nichols, 1976, 1980). Even college calculus students have misconceptions about the true meaning of the equal sign (Clement, Narode, & Rosnick, 1981).

Kieran (1981) reviewed research addressing how students interpret the equal sign and uncovered that students, at all levels of education, lack awareness of its equivalence role. Students in high school and college tend to be more accepting of the equal sign's symbolism for equivalence, however they still described the sign in terms of an operator symbol, with an operation on the left side and a result on the right. Carpenter, Levi, and Farnsworth (2000) further support Kieran's conclusions by noting that elementary students believe the number immediately to the right of an equal sign needs be the answer to the calculation on the left hand side. For example, students filled in the number sentence  $8 + 4 = \square + 5$  with 12 or 17.

According to Carpenter et al. (2000), correct interpretation of the equal sign is essential to the learning of algebra, because algebraic reasoning is based on students' ability to fully understand equality and appropriately use the equal sign for expressing generalizations. For

example, the ability to manipulate and solve equations requires students to understand that the two sides of an equation are equivalent expressions and that every equation can be replaced by an equivalent equation (Kieran, 1981). However, Steinberg, Sleeman, and Ktorza (1990) showed that eighth- and ninth-grade algebra students have a weak understanding of equivalent equations.

Kieran (1981) believes that in order to construct meaning while learning algebra, the notion of the equal sign needs to be expanded while working with arithmetic equalities prior to the introduction of algebra. If this notion were built from students' arithmetic knowledge, the students could acquire an intuitive understanding of the meaning of an equation and gradually transform their understanding into that required for algebra. Similarly, Booth (1986) notes that in arithmetic the equal sign should not be read as "makes", as in "2 plus 3 makes 5" (Booth, 1986), but instead as "2 plus 3 is equivalent to 5", addressing set cardinality.

### *Symbolism*

Unfortunately, the equal sign is not the only symbol whose use in arithmetic is inconsistent with its meaning in algebra (Kieran 1992; Küchemann, 1981). In arithmetic, both the equal and the plus sign are typically interpreted as actions to be performed (Behr et al., 1976, 1980); however, this is not how they are used in algebra. In arithmetic, the plus sign becomes a signal to students to conjoin two terms together (as in  $2 + \frac{1}{2} = 2\frac{1}{2}$ ). However, in algebra,  $2 + x$  is not equal to  $2x$  (Booth, 1986). Both beginning and intermediate algebra students have been found to misinterpret the concatenation of numbers and letters ( $4a$ ) as addition ( $4 + a$ ) instead of multiplication ( $4 \times a$ ) (Kieran, 1988).

To avoid this confusion, Booth (1984) argues that the underlying structure of algebra needs to be exposed to students while working with arithmetic, prior to learning algebra. For example, students are trained throughout arithmetic that solutions are presented in the form of a

single term ( $2 + 5$  is not an acceptable answer). Therefore, students believe that signs such as  $+$  and  $-$  cannot be left in an algebraic answer (such as  $3 + a$ ). This leads to the misuse of concatenation ( $3a$ ) to create an answer that is a single term (Booth, 1988). According to Booth (1984), elementary students should be taught to recognize that the total number of items in two sets containing six and nine objects, respectively, can be written as  $6 + 9$  (rather than 15). This will allow them to see how  $a + b$  represents the total number of items in two sets (containing  $a$  and  $b$  items) and can be treated as a single object and valid answer in algebra (Watson, 1990).

Symbolism is mentioned in a substantial portion of the research addressing algebraic understanding and misconceptions (Behr et al., 1976, 1980; Booth, 1984, 1986; Kieran, 1992; Küchemann, 1981); however, is not directly addressed within the Readiness Indicators (Bottoms, 2003). Similarly, letter usage is cited in a great deal of algebra research (Booth, 1984, 1986, 1988; Küchemann, 1978, 1981; Macgregor & Stacey, 1997, Sleeman, 1984; Usiskin, 1988; Watson, 1990); yet, the Readiness Indicators do not directly address this issue either (Bottoms, 2003).

### *Letter Usage*

The transition from arithmetic to algebra can be troublesome for students not only due to symbol confusion, but also because it is where students are first introduced to the usage of letters in mathematics. This new algebraic notation causes difficulties for many students (Küchemann, 1978, 1981; Macgregor & Stacey, 1997, Sleeman, 1984). According to Watson (1990), variable introduction should be based upon pattern generalization. Children should first learn how to find and record patterns and write pattern-rules in words. Eventually they will seek more concise ways of writing rules. At this time, the introduction of variables will make sense and be

appreciated by the student. The extension and generalization of patterns are also noted in Readiness Indicator number 11 (Bottoms, 2003).

Research shows that novice algebra students do not understand the meaning of letters and commonly interpret letters as standing for objects or words (Macgregor & Stacey, 1997). Even once students are able to accept that letters are standing for numbers, they have a tendency to associate letters with their positions in the alphabet (Watson, 1990) and do not understand that multiple occurrences of the same letter represent the same number (Kieran, 1988). After these misconceptions are addressed, students still view the letters as representing specific unknown values, as in  $3 + x = 8$ , rather than for “numbers in general”, as in  $x + y = y + x$  (Booth, 1986). Küchemann (1978, 1981) found that only a very small percentage of students, ages 13-15, were able to consider a letter as a generalized number. Even fewer were able to interpret a letter as a variable. The majority of the students in Küchemann’s studies treated the letters as concrete objects or just ignored them completely.

Macgregor and Stacey (1997) investigated the origins of students’ misinterpretations of letter usage in algebra, throughout a series of studies involving approximately 2000 students, ages 11-15, across 24 Australian schools. This research uncovered that new content students were learning in mathematics and other subjects (such as computer programming) was interfering with their comprehension of algebraic notation. For example, students combined numbers and letters in algebra using rules from the roman numeration system; some followed the conventions behind writing chemical combinations in chemistry. In fact, Macgregor and Stacey argue that any alternative letter association can produce misconceptions in students’ understanding of algebraic notation. Even the use of letters as a numbering schema in textbooks can cause students to relate letters with their numerical positions in the alphabet.

Misconceptions were also found to be product of misleading teaching materials. When Macgregor and Stacey (1997) tested students across three schools, multiple times throughout a 13-month period, results showed that students in one school had marked difficulty with letter usage in algebra and persistently misinterpreted letters as abbreviated words or labels for objects. However, in the other two schools, only two instances of letters used as abbreviated words were found in the first test and none after that. It was later realized that teaching materials at the latter two schools only used letters to stand for unknown numbers; whereas those of the first school were found to explicitly present letters as abbreviated words (for example  $4d$  could mean “four dogs”).

This research supports Booth (1984, 1986, 1988), who argues that student difficulties in beginning algebra result from the inconsistent usage of letters in arithmetic and algebra. In arithmetic, letters such as “m” and “c” are used as labels to represent meters and cents, not the number of meters or the number of cents, as they would in algebra (Booth, 1988). Teachers read the equation “ $a = l \times w$ ” as “area equals length times width” (Booth, 1986); yet, they are surprised when students claim that the  $y$  in  $5y + 3$  could stand for yachts or yams, but must represent something that starts with a  $y$  (Booth, 1984).

Furthermore, conversions stated  $6m = 600cm$  are read, “6 meters are equivalent to 600 centimeters”. Students use this knowledge to read algebraic equations such as “ $6P = S$ ” as 6 professors are equal to one student (Booth, 1986). Intuitively this implies that there are 6 times as many professors as there are students. However, algebraically this equation is representing the exact opposite. This convention could cause students to incorrectly translate word sentences into algebraic equations. In the reverse of the task above, namely symbolizing that there are six times as many students as professors, the most common error is writing the equation “ $6S = P$ ”, known

as a reversal (Wollman, 1983). This translation, however, would make sense to the student who reads it as a conversion statement, “six students are equal to one professor”.

### *Equation Writing*

Translational errors have been identified throughout a variety of equation writing tasks (Clement, Narode, & Rosnick, 1981). A study including 150 freshmen engineering students noted student difficulties in writing equations from data tables. In fact 51% of the students were unable to generate an equation that was being modeled by a table of data. Here, Clement et al. noted the aforementioned misconception of equality, in addition to the occurrence of reversal errors.

Since reversal errors are so common, Wollman (1983) investigated the actions college students take after they write an equation that is reversed. According to Wollman, students lack the ability (or thought) to check their answers in a meaningful way; this inability or negligence is a key component of students’ performance in algebra. He suggests that students learn to ask themselves questions regarding the equations that they write. Upon investigation, not one student in Wollman’s six studies could remember being taught how to check the meaning of an equation against the meaning of the sentence it was created from. However, once the students were prompted to think about the equations they had written, many were able to produce correct equations or fix their incorrect ones. Perhaps if the practice of answer checking were taught prior to algebra, it would become a natural part of students’ algebraic reasoning and help them in translating various data into algebraic form.

With tools like these, teachers could help strengthen students’ fluency in writing equations, a key component of Readiness Indicator number 10. In fact, the SREB acknowledges many of the identified areas of difficulty within this one indicator, which states that students

need to be able to write, simplify and solve algebraic equations using substitution, the order of operations, the properties of operations and the properties of equality (Bottoms, 2003).

### *Functions*

The SREB also claims that in order to be prepared for Algebra I, students need to understand and be able to represent functions algebraically and graphically, in Readiness Indicator number 12 (Bottoms, 2003). Not only is the concept of a functional relation between two variables a central concept in prealgebra courses, according to Brenner, et al. (1995) translating and applying mathematical representations of functional relations are two cognitive skills that are essential for success in algebraic reasoning. Yet, functions are notoriously difficult for many students to understand (Brenner et al., 1995).

One specific difficulty found among ninth and tenth grade students, who had studied general and linear functions, involved using vocabulary terms associated with functions: preimage, image, domain, range, and image set (Markovits, Eylon, & Bruckheimer, 1988). Students also struggled with certain types of functions, such as constant functions and functions whose graphical representations are disconnected. A common misconception was that every function is a linear function.

According to Markovits et al. (1988), students of lower ability find it easier to handle situations involving functions that are given within a story versus those that are only presented mathematically. Although, it should be noted that much research has discussed difficulties that students encounter while solving word problems (Booth, 1981; Chaiklin, 1989; Clement, 1982; Kieran, 1992; Stacey & MacGregor, 2000). Additionally, Markovits et al. (1988) concluded that students have an easier time handling functions that are given in graphical form versus those in algebraic form. This result implies that the development of graphing capabilities needs to

precede the learning of functions. Similarly, Readiness Indicator number 8 claims that, prior to Algebra I, students should be able to gather, organize, display and interpret data (Bottoms, 2003), that is be fluent with graphs and tables. However, graphing has been specifically identified as a concept that causes problems for algebra students (Brenner, et al., 1995, Chazan & Yerushalmy, 2003; Kieran, 1992; Leinhardt, Zaslavsky, & Stein, 1990).

### *Geometry*

Readiness Indicators numbers 5, 6, and 7 address geometric concepts including the ability to draw and classify geometric figures, measure length, find perimeter, area, surface area and volume, and use the Pythagorean relationship (Bottoms, 2003). Although the current review was not exhaustive, no research-based literature specifically identified geometric skills as prerequisite knowledge or cause of misconception in algebra. However, geometric concepts including area and perimeter have appeared in research investigating algebraic understandings (Booth, 1984; Kieran, 1992; Küchemann, 1978, 1981; Miller & Smith, 1994).

In one such study, Booth (1984) used an item from the Concepts in Secondary Mathematics and Science (CSMS) assessment that involved having students find the area of a rectangle. The rectangle had a height of 7 units, while its length was subdivided into two portions, namely 3 and  $f$ . Booth's interviews showed that students had a good understanding of area and could describe their method for finding area verbally; but when the dimensions included variables, students were not able to correctly symbolize their methods or answers. Perhaps basic geometry skills could be used as a foundation to help students build a better understanding of algebra. Additional research is needed to support this idea, along with the claims stated within Readiness Indicators number 5, 6, and 7.

## Conclusion

Through decades of research on algebraic understanding, researchers, teachers, and curriculum experts have found many content areas that contribute to students' abilities to succeed in algebra. The SREB created a list of 12 algebra-specific Readiness Indicators, which identify skills essential for middle school students to be prepared for Algebra I (Bottoms, 2003).

1. Read, write, compare, order and represent in a variety of forms: integers, fractions, decimals, percents, and numbers written in scientific and exponential notation.
2. Compute (add, subtract, multiply and divide) fluently with integers, fractions, decimals, percents, and numbers written in scientific notation and exponential form, with and without technology.
3. Determine the greatest common factor, least common multiple and prime factorization of numbers.
4. Write and use ratios, rates and proportions to describe situations and solve problems.
5. Draw with appropriate tools and classify different types of geometric figures using their properties.
6. Measure length with appropriate tools and find perimeter, area, surface area and volume using appropriate units, techniques, formulas and levels of accuracy.
7. Understand and use the Pythagorean relationship to solve problems.
8. Gather, organize, display and interpret data.
9. Determine the number of ways an event can occur and the associated probabilities.

10. Write, simplify and solve algebraic equations using substitution, the order of operations, the properties of operations and the properties of equality.
11. Represent, analyze, extend and generalize a variety of patterns.
12. Understand and represent functions algebraically and graphically.

The Readiness Indicators act as a guide for teachers, principals, and curriculum planners to increase student success in algebra (Bottoms, 2003). Eight of the 12 Readiness Indicators, namely numbers 1, 2, 3, 4, 8, 10, 11, and 12, were at least partially, if not fully, supported by the research-based literature examined in the current review. The remaining 4 Readiness Indicators, numbers 5, 6, 7, and 9, were not clearly identified by the research as prerequisite knowledge or cause of misconception in algebra. Therefore, further research is needed to investigate whether the content areas addressed in these specified Readiness Indicators are in fact prerequisite for successful learning of algebra.

## References

- Behr, M., Erlwanger, S., & Nichols, E. (1976) *How children view equality sentences* (PMDC Technical Report No. 3). Tallahassee: Florida State University. (ERIC Document Reproduction Service No: ED144802)
- Behr, M., Erlwanger, S., & Nichols, E. (1980). How children view the equals sign. *Mathematics Teaching, 92*, 13-15.
- Booth, L. R. (1981). Child-methods in secondary mathematics. *Educational Studies in Mathematics, 12*, 29-41.
- Booth, L. R. (1984). *Algebra: Children's strategies and errors*. Windsor, UK: NFER-Nelson.
- Booth, L. R. (1986). Difficulties in algebra. *Australian Mathematics Teacher, 42*(3), 2-4.
- Booth, L. R. (1988). Children's difficulties in beginning algebra. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12* (1988 Yearbook, pp. 20-32). Reston, VA: National Council of Teachers of Mathematics.
- Bottoms, G. (2003). *Getting students ready for Algebra I: What middle grades students need to know and be able to do* (Report No. 02V52). Atlanta, GA: Southern Regional Education Board. (ERIC Document Reproduction Service No. ED476617)
- Brenner, M. E., Brar, T., Durán, R., Mayer, R. E., Moseley, B., Smith, B. R., & Webb, D. (1995, October). *The role of multiple representations in learning algebra*. Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus, OH. (ERIC Document Reproduction Service No. ED391659)

- Carpenter, T. P., Levi, L., & Farnsworth, V. (2000). Building a foundation for learning algebra in the elementary grades. *In Brief, 1*(2). (ERIC Document Reproduction Service No: ED 449015)
- Chaiklin, S. (1989). Cognitive studies of algebra problem solving and learning. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 93-114). Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Lawrence Erlbaum.
- Chazan, D., & Yerushalmy, M. (2003). On appreciating the cognitive complexity of school algebra: Research on algebra learning and directions of curricular change. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 123-135). Reston, VA: National Council of Teachers of Mathematics.
- Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education, 13*, 16-30.
- Clement, J., Narode, R., & Rosnick, P. (1981). Intuitive misconceptions in algebra as a source of math anxiety. *Focus on Learning Problems in Mathematics, 3*(4), 36-45.
- Davis, R. B. (1995). Why are they changing school algebra and who's doing it? *Journal of Mathematical Behavior, 14*, 1-3.
- Falkner, K. P., Levi, L., & Carpenter, T. P. (1999). Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics, 6*(4), 232-236.
- Fey, J. T. (1989). School algebra for the year 2000. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 199-213). Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Lawrence Erlbaum.

- Gallardo, A. (2002). The extension of the natural-number domain to the integers in the transition from arithmetic to algebra. *Educational Studies in Mathematics*, 49, 171-192.
- Herscovics, N. & Kieran, C. (1980). Constructing meaning for the concept of equation. *Mathematics Teacher*, 73, 572-580.
- Kieran, C. (1979). Children's operational thinking within the context of bracketing and the order of operations. In D. Tall (Ed.), *Proceedings of the Third International Conference for the Psychology of Mathematics Education* (pp. 128-133). Coventry, England: Warwick University, Mathematics Education Research Centre.
- Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, 12, 317-326.
- Kieran, C. (1988). The equation-solving errors of novice and intermediate algebra students. In L. Streefland (Ed.), *Proceedings of the Ninth International Conference for the Psychology of Mathematics Education* (pp. 141-146). Noordwijkerhout, The Netherlands: State University of Utrecht, Research Group on Mathematics Education and Educational Computer Centre.
- Kieran, C. (1989). The early learning of algebra: A structural perspective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 33-56). Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Lawrence Erlbaum.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). New York: Macmillan.

- Küchemann, D. E. (1978). Children's understanding of numerical variables. *Mathematics in School*, 7(4), 23-26.
- Küchemann, D. E. (1981). Algebra. In K. M. Hart (Ed.), *Children's understanding of mathematics: 11-16*, (pp. 102-119). London: John Murray.
- Leinhardt, G., Zaslavsky, O., Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, 60, 1-64.
- Linchevski, L. (1995). Algebra with numbers and arithmetic with letters: A definition of pre-algebra. *Journal of Mathematical Behavior*, 14, 113-120.
- Macgregor, M., & Stacey, K. (1997). Students' understanding of algebraic notation: 11-15. *Educational Studies in Mathematics*, 33, 1-19.
- Markovits, Z., Eylon, B.-S., & Bruckheimer, M. (1988). Difficulties students have with the function concept. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12* (1988 Yearbook, pp. 43-60). Reston, VA: National Council of Teachers of Mathematics.
- Miller, C. A., & Smith, B. D. (1994). Assessment of prerequisite mathematics vocabulary terms for intermediate and college algebra. *Focus on Learning Problems in Mathematics*, 16(2), 39-50.
- Moses, R. P. (1994). Remarks on the struggle for citizenship and math/science literacy. *Journal of Mathematical Behavior*, 13(1), 107-11.
- Moses, R. P., Kamii, M., Swap, S. M., & Howard, J. (1989). The algebra project: Organizing in the spirit of Ella. *Harvard Educational Review*, 59, 423-443.
- Picciotto, H., & Wah, A. (1993). A new algebra: Tools, themes, concepts. *Journal of Mathematical Behavior*, 12, 19-42.

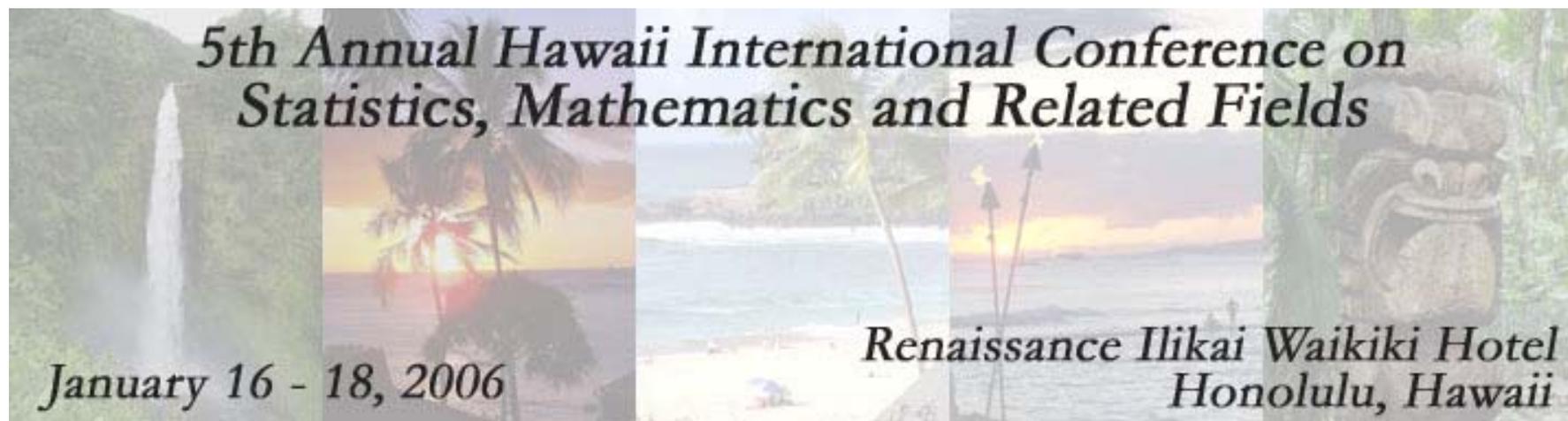
- Pinchback, C. L. (1991). Types of errors exhibited in a remedial mathematics course. *Focus on Learning Problems in Mathematics*, 13(2), 53-62.
- Post, T. R., Behr, M. J., & Lesh, R. (1988). Proportionality and the development of prealgebra understandings. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12* (1988 Yearbook, pp. 78-90). Reston, VA: National Council of Teachers of Mathematics.
- Rachlin, S. L. (1989). The research agenda in algebra: A curriculum development perspective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 257-265). Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Lawrence Erlbaum.
- Rotman, J. W. (1991, November). *Arithmetic: Prerequisite to algebra?* Paper presented at the Annual Convention of the American Mathematical Association of Two-Year Colleges, Seattle, WA. (ERIC Document Reproduction Service No. ED338279)
- Sleeman, D. H. (1984). An attempt to understand students' understanding of basic algebra. *Cognitive Science*, 8, 387-412.
- Stacey, K., & MacGregor, M. (2000). Learning the algebraic method of solving problems. *Journal of Mathematical Behavior*, 18, 149-167.
- Steinberg, R. M., Sleeman, D. H., & Ktorza, D. (1990). Algebra students' knowledge of equivalence of fractions. *Journal for Research in Mathematics Education*, 22(2), 112-121.
- Usiskin, Z. (1987). Why elementary algebra can, should and must be an eighth-grade course for average students. *Mathematics Teacher*, 80, 428-438.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. F. Coxford & A. P. Schulte (Eds.), *The ideas of algebra, K-12* (1988 Yearbook, pp. 8-19). Reston, VA: National Council of Teachers of Mathematics.

Watson, J. (1990). Research for teaching: Learning and teaching algebra. *Australian Mathematics Teacher*, 46(3), 12-14.

Wollman, W. (1983). Determining the sources of error in a translation from sentence to equation. *Journal for Research in Mathematics Education*, 14, 169-181.

Wu, H. (2001). How to prepare students for algebra. *American Educator*, 25(2), 10-17.



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